The Length of a Fish

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MATH 2414

Background

Being able to model and predict fish growth has been extremely important to fisheries scientists for many years. However, being able to accurately model fish growth was not possible until 1934, when Austrian biologist Ludwig von Bertalanffy published the von Bertalanffy growth equation.

Figure 1: Ludwig Von Bertalanffy

t = Age(years)

K = The growth coefficient of that fish

L(0) = The length of that fish right when it was born

L_∞ = The longest length that fish could ever grow to be (asymptotic length)

$$\frac{dL}{dt} = k(L_\infty - L)$$

There are many other revised equations out there today however, we will only be focusing on this one because it is the most popular and widely used.

Finding the Equation

Take derivative using product rule:

$(Le^{kt})'$

$e^{kt}\ \frac{dL}{dt} + e^{kt}\ \text{kL}$

$$\frac{dL}{dt} = \text{k}(L_\infty - \text{L})$$

$$\frac{dL}{dt} = \text{k}L_\infty - \text{kL}$$

$$\frac{dL}{dt} + \text{kL} = \text{k}L_\infty$$

$$e^{kt}\ \frac{dL}{dt} + e^{kt}\ \text{kL} = e^{kt}\ \text{k}L_\infty$$

$$\int (Le^{kt})' = \int e^{kt}\ \text{k}L_\infty$$

$$Le^{kt} = L_\infty e^{kt} + \text{C}$$

$$L = \frac{1}{e^{kt}}(L_\infty e^{kt} + \text{C})$$

$$L = L_\infty + Ce^{-kt}$$

$$L(\text{t}) = L_\infty + (\text{L}(0) - L_\infty)e^{-kt}$$

$$L(\text{t}) = L_\infty - (L_\infty\ - \text{L}(0))\ e^{-kt}$$

Finding the integrating factor:

$e^{\int k\ dt}$

e^{kt}

Finding C:

$L(0) = L_\infty + \text{C}e^0$

$L(0) = L_\infty + \text{C}$

$\text{C} = L(0) - L_\infty$

t = Age(years)

K = The growth coefficient of that fish

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Example

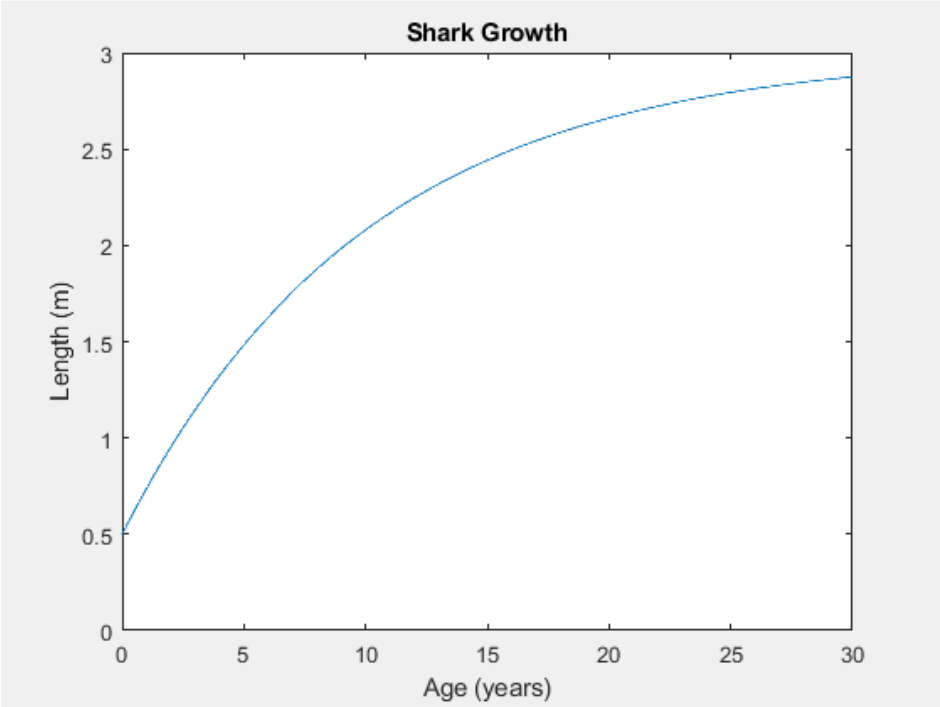
- Let's say most sharks can only grow to be around 3 m and they grow around 10 cm a year. If it was born at a size of .5 m, what is its size of the shark at an age of 8 years old?

$L(t) = L_\infty - (L_\infty - L(0))\, e^{-kt}$

$L(8) = 3 - (3 - .5)\, e^{-.10(8)}$

$L(8) = 1.89$ m

Therefore, the shark will be around 1.89 m long at the age of 8.



Conclusion

Before starting this project, I never realized how many factors go into determining the length of a fish at a given time. I also never knew how much time, work, and effort was put into creating and revising this equation. On top of that, I learned a special method that can be used to help solve differential equations. With the use of modeling fish growth rapidly increasing in this day and age, there is more of a reason to share why this is such an important topic. Overall, I found the von Bertalanffy growth equation quite fascinating, and I hope you did too.

Work Cited

“Figure 1: Ludwig Von Bertalanffy.” Ludwig Von Bertalanffy, 545550393541748825.weebly.com/ludwig-von-bertalanffy.html.